

Gravity and Yang-Mills Amplitude Relations

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Using only general features of the S-matrix and quantum field theory, we prove by induction the Kawai-Lewellen-Tye relations that link products of gauge theory amplitudes to gravity amplitudes at tree level. As a bonus of our analysis, we provide a novel and more symmetric form of these relations. We also establish an infinite tower of new identities between amplitudes in gauge theories.

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Introduction. A most astonishing connection between Einstein gravity and color-ordered Yang-Mills tree amplitudes is provided by the Kawai-Lewellen-Tye (KLT) relations [1]. The infinite sequence of these KLT-relations was discovered as a consequence of factorizing a closed-string amplitude into a product of open-string amplitudes and subsequently taking the field theory limit. For a nice introduction to the details of this, see, *e.g.*, ref. [2] and references therein. At the quantum field theory level the KLT-relations present a mysterious puzzle, since neither the Einstein-Hilbert nor the Yang-Mills Lagrangians provide any hints at their origin. In fact, the two theories appear to be very dissimilar in structure. The gravity Lagrangian yields perturbative Feynman rules with an infinite series of higher-point graviton vertices while the Yang-Mills Lagrangian terminates at four-point vertices. The gravity Lagrangian has general coordinate covariance, while Yang-Mills theory has local gauge invariance. It could seem that something close to a miracle would be required to relate the two associated S-matrices. It is one of the great achievements of string theory that it inspires a re-organization of the perturbative expansions that sheds completely new light on this. In a related development, it has been shown in ref. [3] how string theory can be used to derive the conjectured Bern-Carrasco-Johansson (BCJ) identities in gauge theories with and without matter [4, 5], see also [6]. While this illustrates again the power of string-based techniques, it also highlights the need for a similar understanding directly at the field theory level. Very recently, the BCJ-relations were proven [7] using only quantum field theory, based on the method of on-shell recursion [8, 9]. There has also been attempts at more conventional ways to understand the KLT-relations at the Lagrangian level [10] through judicious choices of gauges. One possibility is a reformulation of the Yang-Mills Lagrangian through the addition of spurious vertices up to infinite order [11]. An alternative path consists in writing the gauge theory amplitudes explicitly in terms of a selected set of pole structures. A squaring relation between gravity and gauge theory poles, conjectured to hold to all orders in ref. [4], can then be proven [11].

In the light of recent progress, we will here take a fresh approach to the KLT-relations. We prove these relations us-

ing only quantum field theory and general properties of the S-matrix [12]. As a spin-off, we uncover a new series of highly non-trivial relations entirely on the gauge theory side. These identities are non-linear and involve products of different helicity configurations of gauge theory amplitudes. We provide a novel form of the KLT-relations as well. It has a higher degree of manifest symmetry than the one previously suggested in the literature [13]. Interestingly, the two different forms are precisely related to each other via the BCJ-relations.

Gravity from Gauge Theory and New Relations between Gauge Theory Amplitudes. We denote an n -point gravity amplitude of fixed helicity by $M_n(1, 2, \dots, n)$ and let $A_n(1, 2, \dots, n)$ and $\tilde{A}_n(1, 2, \dots, n)$ stand for n -point color-ordered gauge theory amplitudes of fixed helicity. Both classes of amplitudes have been stripped of coupling constants since it is trivial to reinstate them. We also denote, as usual, $s_{12\dots i} \equiv (p_1 + \dots + p_i)^2$. Our two main results are

$$M_n(1, 2, \dots, n) = (-1)^n \sum_{\gamma, \beta} \frac{\tilde{A}_n(n, \gamma_{2,n-1}, 1) \mathcal{S}[\gamma_{2,n-1} | \beta_{2,n-1}] A_n(1, \beta_{2,n-1}, n)}{s_{123\dots(n-1)}}, \quad (1)$$

$$0 = \sum_{\gamma, \beta} \frac{\tilde{A}_n(n, \gamma_{2,j^+,n-1}, 1) \mathcal{S}[\gamma_{2,n-1} | \beta_{2,n-1}] A_n(1, \beta_{2,j^-,n-1}, n)}{s_{123\dots(n-1)}}, \quad (2)$$

both of which will be proven by induction. We have chosen one arbitrary external leg j to have opposite helicity in eq. (2). The ordering of legs $2, 3, \dots, n-1$ in the amplitude A_n is denoted $\gamma_{2,n-1}$ and $\gamma_{2,j^\pm,n-1}$, where j^\pm indicates that leg j has been assigned a specific helicity. We note here since we will use it later that it is also possible that the assigned helicity leg j^\pm is either leg 1 or n . The corresponding ordering in the amplitude A_n is denoted by $\beta_{2,n-1}$ and $\beta_{2,j^\pm,n-1}$ and we sum over all permutations of both γ and β . The function \mathcal{S} is defined by

$$\mathcal{S}[i_1, \dots, i_k | j_1, \dots, j_k] \equiv \prod_{t=1}^k (s_{i_t+1} + \sum_{q>t}^k \theta(i_t, i_q) s_{i_t i_q}), \quad (3)$$

where $\theta(i_a, i_b)$ is 0 if i_a sequentially comes before i_b in $\{j_1, \dots, j_k\}$, and otherwise it is 1. To illustrate, $\mathcal{S}[2|2] = s_{12}$, $\mathcal{S}[23|23] = s_{12}s_{13}$, $\mathcal{S}[23|32] = s_{13}(s_{12} + s_{23})$, and so on.

The function \mathcal{S} has some nice properties that all follow from its definition (3) by use of elementary algebra. These properties will play an essential role in what follows. In particular,

$$\mathcal{S}[i_1, \dots, i_k | j_1, \dots, j_k] = \mathcal{S}[j_k, \dots, j_1 | i_k, \dots, i_1], \quad (4)$$

which ensures that the expressions (1) and (2) are completely symmetric in \tilde{A}_n and A_n . It is also convenient to introduce an auxiliary function,

$$\mathcal{S}_P[i_1, \dots, i_k | j_1, \dots, j_k] = \prod_{t=1}^k (s_{i_t P} + \sum_{q>t}^k \theta(i_t, i_q) s_{i_t i_q}), \quad (5)$$

which coincides with \mathcal{S} except for the fact that the momentum of leg 1 has been replaced by a sum of momenta, $P \equiv p_1 + p_2 + \dots + p_m$ with $P^2 = 0$, which not necessarily involves any of the momenta in the brackets. We point out that one has the factorization

$$\mathcal{S}[\gamma_{q+1,k} \sigma_{2,q} | \alpha_{2,q} \beta_{q+1,k}] = \mathcal{S}[\sigma_{2,q} | \alpha_{2,q}] \times \mathcal{S}_P[\gamma_{q+1,k} | \beta_{q+1,k}], \quad (6)$$

with $P = p_1 + p_2 + \dots + p_q$.

We now note the following:

- Eq. (1) provides the general n -point result for the field theory limit of the KLT-relations [1].
- Eq. (2) provides a new set of identities between gauge theory amplitudes of different helicity configurations.

An unusual property of the expressions (1) and (2) is that they appear to be singular on-shell. However, the singularity due to $s_{12\dots(n-1)}$ is only apparent: It is always cancelled by a similar factor in the numerator. This will be explained below.

A different form of the KLT-relations was conjectured in ref. [13]. Our new expression (1), which keeps only two legs fixed while summing over all permutations of the remaining legs, is more symmetric and therefore more convenient for our purpose.

Proof by induction: We will treat the cases (1) and (2) in parallel. To handle the apparent singularity of $s_{12\dots(n-1)}$ we need to regularize both expressions (1) and (2). We could choose the following regularization:

$$\begin{aligned} p_1 &\rightarrow p_1 - xq, \\ p_n &\rightarrow p_n + xq, \end{aligned} \quad (7)$$

with a parameter x , $p_1 \cdot q = 0$ and $q^2 = 0$, but $q \cdot p_n \neq 0$. This keeps $p_1^2 = 0$, respects overall momentum conservation, but makes $p_n^2 = s_{12\dots(n-1)} \neq 0$. We recover the physical amplitudes in the limit $x \rightarrow 0$.

Before proceeding further, we make a few more remarks regarding the regularization (7) and how one cancels the pole $s_{12\dots(n-1)}$. Interestingly, the numerators of (1) and (2) vanish on-shell precisely because of BCJ-relations. In detail, for each $\gamma_{2,n-1}$ permutation,

$$\sum_{\beta} \mathcal{S}[\gamma_{2,n-1} | \beta_{2,n-1}] A_n(1, \beta_{2,n-1}, n) = 0, \quad (8)$$

is in general a combination of BCJ-relations. One can write an analogous relation for \tilde{A}_n by means of a γ -permutation sum. Once the full numerators in (1) and (2) are regularized according to, for instance, eq. (7), they do not vanish. Lifting the regularization from terms that remain finite in the $x \rightarrow 0$ limit, one can systematically exploit on-shell BCJ-relations to factor out the needed factor of $p_n^2 = s_{12\dots(n-1)}$ which cancels the would-be pole. Afterwards the limit can safely be taken in all remaining terms. This reduction, however, destroys the larger manifest permutation symmetry of (1) and (2), and the reduced expression is therefore not the most convenient form for a BCFW-analysis [8, 9]. We have checked up to $n = 8$ that the reduced expression agrees with the general formula suggested in ref. [13]. More details on this will be presented elsewhere [14].

When $n = 3$ both eq. (1) and eq. (2) hold trivially. The right-hand side of both equations becomes, after removing the regularization, $-\tilde{A}_3(3, 2, 1) A_3(1, 2, 3)$. On-shell, for real momenta, these 3-point amplitudes vanish, and both identities are satisfied. For on-shell complex momenta the relation (1) reads $M_3(1, 2, 3) = -\tilde{A}_3(3, 2, 1) A_3(1, 2, 3)$, which is indeed the correct three-graviton amplitude [12] (both sides vanish when all helicities are equal). For $n = 4$ the right-hand side becomes

$$\begin{aligned} &s_{12} \tilde{A}_4(4, 3, 2, 1) \frac{(s_{13} + s_{23}) A_4(1, 2, 3, 4) + s_{13} A_4(1, 3, 2, 4)}{s_{123}} + \\ &s_{13} \tilde{A}_4(4, 2, 3, 1) \frac{(s_{12} + s_{23}) A_4(1, 3, 2, 4) + s_{12} A_4(1, 2, 3, 4)}{s_{123}}, \end{aligned} \quad (9)$$

where we have collected pieces so that the mentioned structure of BCJ-relations appears in the numerator. We can then take the limit $x \rightarrow 0$ in the two terms $s_{12} \tilde{A}_4(4, 3, 2, 1)$ and $s_{13} \tilde{A}_4(4, 2, 3, 1)$ separately, use the on-shell BCJ-relation $s_{12} \tilde{A}_4(4, 3, 2, 1) = s_{13} \tilde{A}_4(4, 2, 3, 1)$ and collect terms to get an overall factor of s_{123} which precisely cancels the denominator. The regularization can then be removed. Doing these steps we are left with

$$s_{12} \tilde{A}_4(4, 3, 2, 1) [A_4(1, 2, 3, 4) + A_4(1, 3, 2, 4)],$$

which by use of standard amplitude relations can be written as the more familiar KLT expression $-s_{12} \tilde{A}_4(1, 2, 3, 4) A_4(1, 2, 4, 3)$.

For the identities to be of interest, we of course take helicities so that the amplitudes are non-vanishing to begin with.

Flipping the helicity of one of the external legs in either \tilde{A}_4 or A_4 will cause those amplitudes to vanish, and eq. (2) is thus trivially satisfied. If we do not flip the helicity of one of the legs, we see by explicit computation that we get the four-graviton amplitude $M_4(1, 2, 3, 4)$ for the chosen helicities.

The origin of the cancellation of the $s_{12\dots(n-1)}$ -pole hinges on the basis of amplitudes being of size $(n-3)!$ [3], while the permutation sums in (1) and (2) keep only *two* legs, 1 and n , fixed. The sums are therefore overcomplete and redundant.

After these preliminary remarks, we are now ready to prove the general relations by induction. We have already demonstrated by explicit computations that the relations hold for both real and complex momenta when $n = 3$ and $n = 4$. We next assume that eq. (1) and eq. (2) both hold for $n - 1$. Doing a BCFW-shift in legs 1 and n , we consider the following contour integral

$$0 = \oint \frac{dz}{z} M_n(z) = M_n(0) + (\text{residues for } z \neq 0). \quad (10)$$

If there should be boundary terms to the integral, they are ignored here. It is known that the n -point gravity amplitudes M_n have sufficiently rapid fall-off at infinity to exclude boundary terms [15–17].

For the $z \neq 0$ residues, we consider separately the following two classes of contributions:

- (A) The pole appears in only one of the amplitudes \tilde{A}_n and A_n .
- (B) The pole appears in both amplitudes \tilde{A}_n and A_n .

We start with the case (A). By symmetry, we need to consider only \tilde{A}_n having the pole. When the pole instead sits in A_n the reasoning is identical. For ease of notation we will omit the explicit writing of $\lim_{x \rightarrow 0}$ in what follows.

Considering eq. (1), the residue of the pole $s_{\hat{1}2\dots k}$ can be calculated from $-\lim_{z \rightarrow z_{12\dots k}} [s_{\hat{1}2\dots k}(z) M_n(z)] / s_{12\dots k}$, where $z_{12\dots k}$ is the z -value that makes $s_{\hat{1}2\dots k}$ go on-shell. We hence get

$$\frac{(-1)^{n+1}}{s_{\hat{1}2\dots n-1}} \sum_{\gamma, \sigma, \beta} \frac{\sum_h \tilde{A}_{n-k+1}(\hat{n}, \gamma, -\hat{P}^h) \tilde{A}_{k+1}(\hat{P}^{-h}, \sigma, \hat{1})}{s_{12\dots k}} \times \mathcal{S}[\gamma\sigma|\beta_{2,n-1}] A_n(\hat{1}, \beta_{2,n-1}, \hat{n}), \quad (11)$$

where we have introduced the short-hand notation $\gamma \equiv \gamma_{k+1,n-1}$ and $\sigma \equiv \sigma_{2,k}$. Now using a factorization analogous to eq. (6) we can rewrite

$$\mathcal{S}[\gamma\sigma|\beta_{2,n-1}] = \mathcal{S}[\sigma|\rho_{2,k}] \times (\text{a factor independent of } \sigma), \quad (12)$$

where $\rho_{2,k}$ denotes the relative ordering of legs $2, 3, \dots, k$ in β . We thus see that eq. (11) contains

$$\sum_{\sigma} \tilde{A}_{k+1}(\hat{P}^{-h}, \sigma, \hat{1}) \mathcal{S}[\sigma|\rho_{2,k}] = 0, \quad (13)$$

which is zero at $z = z_{12\dots k}$, as indicated. It is important for this argument that A does not have a pole at $s_{12\dots k}$ since such a pole could cancel the above zero. We hereby conclude that all terms coming from the class (A) above will not contribute to the residues. Going through the analogous argument for eq. (2) we conclude similarly about the case (A) there.

We now turn to the class (B). Again we consider first eq. (1) and the $s_{12\dots k}$ pole contribution. Here both \tilde{A} and A have the pole. Similar to the short-hand notation of γ and σ above, we will also introduce $\beta \equiv \beta_{k+1,n-1}$ and $\alpha \equiv \alpha_{2,k}$.

When both amplitudes have the pole, the residue takes the following form (we have in the two equations below suppressed the subscript index on A and \tilde{A} to avoid unnecessary cluttering of the expressions)

$$\frac{(-1)^{n+1}}{s_{\hat{1}2\dots(n-1)}} \sum_{\gamma, \beta, \sigma, \alpha} \left[\frac{\sum_h \tilde{A}(\hat{n}, \gamma, \hat{P}^{-h}) \tilde{A}(-\hat{P}^h, \sigma, \hat{1})}{s_{12\dots k}} \right] \mathcal{S}[\gamma\sigma|\alpha\beta] \left[\frac{\sum_h A(\hat{1}, \alpha, -\hat{P}^h) A(\hat{P}^{-h}, \beta, \hat{n})}{s_{\hat{1}2\dots k}} \right], \quad (14)$$

where one of the shifted $s_{\hat{1}2\dots k}$ poles have been replaced by an unshifted pole $s_{12\dots k}$ from calculating the single-pole residues. We now wish to collect pieces so that lower-point amplitude combinations $\tilde{A}_k A_k$ appear in forms ready for a BCFW-interpretation at these lower points. Noting that $s_{\hat{1}2\dots n-1} = s_{\hat{P}_{k+1\dots n-1}}$, and using $\mathcal{S}[\gamma\sigma|\alpha\beta] = \mathcal{S}[\sigma|\alpha] \times \mathcal{S}_{\hat{P}}[\gamma|\beta]$, this can be achieved by writing the above contribution as

$$\frac{(-1)^{n+1}}{s_{12\dots k}} \sum_h \left[\left(\sum_{\sigma, \alpha} \frac{\tilde{A}(-\hat{P}^h, \sigma, \hat{1}) \mathcal{S}[\sigma|\alpha] A(\hat{1}, \alpha, -\hat{P}^h)}{s_{\hat{1}2\dots k}} \right) \left(\sum_{\gamma, \beta} \frac{\tilde{A}(\hat{n}, \gamma, \hat{P}^{-h}) \mathcal{S}_{\hat{P}}[\gamma|\beta] A(\hat{P}^{-h}, \beta, \hat{n})}{s_{\hat{P}_{k+1\dots(n-1)}}} \right) \right] + (h, -h), \quad (15)$$

where $(h, -h)$ means the same expression again, but with the $(-\hat{P}^h, -\hat{P}^h)$ in the first parenthesis and $(\hat{P}^{-h}, \hat{P}^{-h})$ in the second parenthesis replaced by $(-\hat{P}^h, -\hat{P}^{-h})$ and $(\hat{P}^h, \hat{P}^{-h})$, respectively. These are *mixed-helicity* terms. The appearance of mixed-helicity terms is what prevents an immediate recombination into lower-point M_n -amplitudes.

Fortunately, the first term of eq. (15) is nothing but the product of two lower-point expressions of eq. (1). We remind the reader that we have suppressed the overall $\lim_{x \rightarrow 0}$ and $\lim_{z \rightarrow z_{12\dots k}}$ at all steps in the derivation, ensuring that both expressions are well-defined, *i.e.*

$$-\frac{\sum_h M_{k+1}(\hat{1}, 2, \dots, k, -\hat{P}^h) M_{n-k+1}(\hat{P}^{-h}, k+1, \dots, \hat{n})}{s_{12\dots k}}. \quad (16)$$

Summing over all the permutations, these pieces precisely build up the amplitude M_n by means of on-shell recursion. The second term of eq. (15) is a mixed-helicity expression, identical to the type of relations given by eq. (2).

The proof of eq. (2) for the mixed-helicity relations follows exactly the same steps as in the derivation of (1) above. The only difference is that in eq. (15) each helicity sum has a part that includes a lower-point mixed-helicity relation. By our induction hypothesis, this is zero. Because of the manifest permutation symmetry in all legs, except for the shifted legs 1 and n , every other contribution to the residue follows from this case by a permutation. The mixed-helicity terms in eq. (15) hence vanish. This therefore concludes our proof by induction of both our new form of the KLT-relations eq. (1) and the new relations between gauge theory amplitudes eq. (2).

Conclusions. We have discovered a new and more symmetric form of the KLT-relations which relate tree-level gravity amplitudes to products of gauge theory amplitudes. In the process we have uncovered a series of non-linear identities among gauge theory amplitudes where helicities are flipped. We have proven by induction both sets of relations using on-shell recursion methods. Our proof does not rely on any other properties of the amplitudes than those provided by quantum field theory and general assumptions about the S-matrix.

We have here concentrated only on the basic identity between pure Yang-Mills theory and gravity because this is the perhaps most startling result. As will be discussed in detail elsewhere [14], one can straightforwardly extend the analysis to include all amplitudes from full supersymmetric multiplets on the gauge theory and gravity sides, *i.e.* $\mathcal{N} = 4$ super Yang-Mills theory and $\mathcal{N} = 8$ supergravity, respectively. This includes analogous relations for amplitudes of mixed particle content in A_n and \tilde{A}_n [18]. The new series of gauge theory identities (2) is only a particular example of a more general series of identities where also more than one pair of helicities can be flipped [14]. Although these new identities have natural interpretations in a KLT-like language, they are nevertheless on a different footing. It would be nice to also understand these new identities in the light of string theory.

In a broader perspective, it should be of interest to understand the significance of the relation between gravity amplitudes and gauge theory amplitudes at loop level as well. There

has very recently been interesting progress in this direction [19]. Also here the method of on-shell recursion may provide new insight.

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